

Performance and Capacity of PAM and PPM UWB Time-Hopping Multiple Access Communications with Receive Diversity

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The error probability and capacity of a time-hopping ultra-wideband (UWB) communication system with receive diversity are investigated. We consider pulse amplitude modulation (PAM) and pulse-position modulation (PPM) over additive white Gaussian channels for a single-user system. A multiuser environment with PPM is also investigated. It is shown that the communication distance and error performance are improved by employing receive diversity. The channel capacity of PPM and PAM is determined subject to the power constraints of FCC part 15 rules to illustrate the relationship between reliable communication distance and signal-to-noise ratio. The error probability with PAM and receive diversity is derived for the single-user case. The error probability and performance bounds with PPM are derived for both the single-user and multiuser cases.

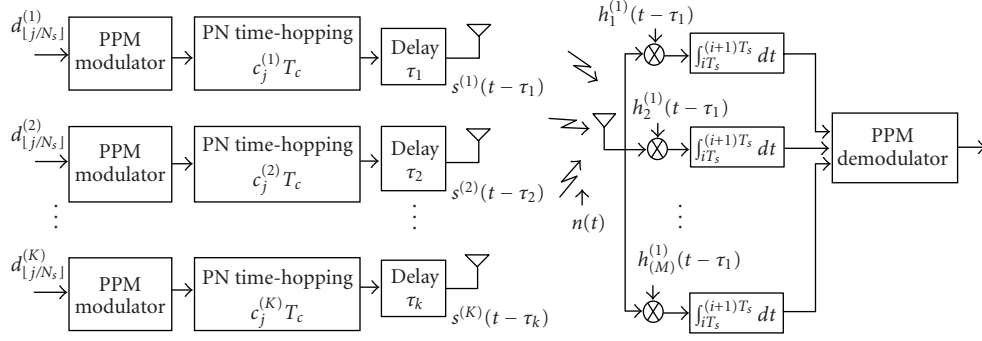
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1. INTRODUCTION

An ultra-wideband (UWB) [1] communication system transmits information using ultrashort impulses that spread the energy of the signal typically from near DC to several GHz. Unlike conventional communication systems, UWB systems operate at baseband, and thus involve no intermediate frequency and no carrier synchronization. UWB theoretically promises a very high data rate by employing a large signal bandwidth. However, due to possible interference to existing communication systems, power spectrum density limitations such as FCC part 15 rules are imposed, which greatly limits the system capabilities. In particular, UWB systems under FCC part 15 rules provide reliable communications only over small to medium distances. Typically pulse amplitude modulation (PAM), pulse-position modulation (PPM), or on/off keying (OOK) modulation is employed. PPM modulation uses the precise collocation of the impulses in time to convey information, while PAM and OOK use amplitude for this purpose. Note that the multipath signal is resolvable down to path delays on the order of a nanosecond or less due to the use of ultrashort impulses. This can be exploited to significantly reduce the effects of fading in a wireless environment.

UWB systems with PAM and PPM modulation have been extensively investigated. In [1, 2, 3, 4], a time-hopping multiple access scheme for UWB systems with PPM was considered. A PPM UWB system over an AWGN channel was considered from the capacity perspective (subject to FCC part 15 rules) in [5, 6]. The performance of a PAM UWB system with a RAKE receiver was investigated in [7, 8] for an indoor wireless channel with multipath interference. An all-digital multiple access system based upon PAM and time-division multiplexing was proposed in [9]. The construction of equal-energy N -orthogonal time-shift-modulated codes was described in [10]. In [11], the effective capacity of a pulse-position hopping code division multiple access (CDMA) system with OOK modulation was analyzed.

In this paper, receive diversity is considered for a time-hopping UWB communication system to improve the error performance, communication distance, and capacity. Receive diversity can be achieved using either a RAKE receiver or multiple receive antennas. Both PAM and PPM are considered for a single-user environment, and a multiuser environment is also investigated for PPM. Channel capacity is investigated subject to FCC part 15 rules to illustrate the relationship between reliable communication distance and

FIGURE 1: System model of a time-hopping multiple access M -ary PPM UWB system.

signal-to-noise ratio (SNR). The exact error probability and performance bounds are derived for additive white Gaussian noise (AWGN) channels with PAM and PPM in a single-user environment, as well as PPM in a multiple access environment. Differentiated Gaussian pulses are used for the multiple access analysis, which can easily be extended to other waveforms.

The remainder of the paper is organized as follows. In Section 2, the system model and construction of the time-hopping PPM and PAM UWB signals are described. The capacity and error probability analysis for PPM and PAM with receive diversity are given in Section 3 for a single-user environment. The relationship between the reliable communication distance and channel capacity subject to FCC part 15 rules is demonstrated. The exact error probability for M -ary PPM and PAM with receive diversity is also presented, and a simple upper bound on the probability of error for PPM is provided. Section 4 presents the capacity and error probability analysis for a multiple access UWB system with receive diversity. Numerical results on capacity and performance with receive diversity are given in Section 5. Finally, Section 6 provides some conclusions.

2. SIGNAL CONSTRUCTION AND THE SYSTEM MODEL

A typical time-hopping format for the output of the k th user in a UWB system is given by [12]

$$s^{(k)}(t) = \sum_{j=-\infty}^{\infty} A_{d_{l_j/N_s}^{(k)}} q(t - jT_f - c_j^{(k)} T_c - \delta_{d_{l_j/N_s}^{(k)}}), \quad (1)$$

where $A^{(k)}$ is the signal amplitude, $q(t)$ represents the transmitted impulse waveform that nominally begins at time zero at the transmitter, and the quantities associated with (k) are transmitter dependent. T_f is the frame time, which is typically a hundred to a thousand times the impulse width resulting in a signal with a very low duty cycle. Each frame is divided into N_h time slots with duration T_c . The pulse-shift pattern $c_j^{(k)}$, $0 \leq c_j^{(k)} \leq N_h$ (also called the time-hopping sequence), is pseudorandom with period T_c . This provides an additional shift in order to avoid catastrophic collisions due to multiple access interference (MAI). The sequence d is the data stream generated by the k th source after channel coding,

and δ is the additional time shift utilized by the N -ary PPM. If $N_s > 1$, a repetition code is introduced, that is, N_s pulses are used to transmit the same information.

For M -ary PPM, we have unit signal amplitude, that is, $A^{(k)} = 1$, so that (1) can be written as

$$s^{(k)}(t) = \sum_{j=-\infty}^{\infty} q(t - jT_f - c_j^{(k)} T_c - \delta_{d_{l_j/N_s}^{(k)}}). \quad (2)$$

For M -ary PAM, we have no additional modulation time shift, that is, $\delta = 0$, and the signal amplitude is defined as $A_m = 2m - 1 - M$, $1 \leq m \leq M$, so that (1) can be written as

$$s^{(k)}(t) = \sum_{j=-\infty}^{\infty} A_{d_{l_j/N_s}^{(k)}} q(t - jT_f - c_j^{(k)} T_c). \quad (3)$$

The received signal can be modeled as the derivative of the transmitted pulses assuming propagation in free space [1]:

$$\begin{aligned} r(t) &= \sum_{l=1}^L \left[\sum_{k=1}^K (s^{(k)}(t - \tau_{lk}))' + w_l(t) \right] \\ &= \sum_{l=1}^L \left[\sum_{k=1}^K \sum_{j=-\infty}^{\infty} A_{d_{l_j/N_s}^{(k)}} p(t - jT_f - c_j^{(k)} T_c \right. \\ &\quad \left. - \delta_{d_{l_j/N_s}^{(k)}} - \tau_{lk}) + w_l(t) \right], \end{aligned} \quad (4)$$

where $w_l(t)$ is AWGN noise with power density $N_0/2$, τ_{lk} is the propagation delay for the k th user, $p(t)$ is the received pulse waveform, and L is the receive diversity order. Note that equal gain combining (EGC) is assumed at the receiver for simplicity. If only one user is present, the optimal receiver for PPM is a bank of M correlation receivers followed by a detector. When more than one link is active in the multiple access system, the optimal PPM receiver has a complex structure that takes advantage of all receiver knowledge regarding the characteristics of the MAI [6]. However, for simplicity, an M -ary correlation receiver is typically used for PPM even when there is more than one active user. For PAM, only one correlation receiver is required for both the single-user and multiuser cases. Figure 1 shows the structure of the correlation receiver of an M -ary PPM UWB system.

3. SINGLE-USER CAPACITY AND ERROR PROBABILITY

3.1. Channel capacity for M -ary PPM over AWGN channels

With a single user active in the system, AWGN is the only source of signal degradation. For simplicity, we further assume for PPM that $\delta \geq T_p$, where T_p is the pulse duration, that is, the M -ary PPM signal constellation consists of a set of M orthogonal signals with equal energy. The vector representation for an M -ary PPM signal is defined as an M -dimensional vector with nonzero value in the m th dimension:

$$\mathbf{s}_m = [0, \dots, 0, \sqrt{E_g}, 0, \dots, 0], \quad (5)$$

where E_g is the average signal energy. Then the analysis in [13, 14] for the capacity of modulated channels for PPM and PAM, and the error probability analysis in [15] for PPM and PAM, can be extended to include receive diversity.

The Shannon capacity for an AWGN channel with continuous-valued inputs and outputs is given by $C = W \log_2(1 + \text{SNR})$, where W is the channel bandwidth. However, a channel with M -ary PPM modulation has discrete-valued inputs and continuous-valued outputs, which imposes an additional constraint on the capacity expression. Let \mathbf{s} be the encoded M -dimensional PPM signal vector input to the channel, and let \mathbf{r}_l be the l th diversity channel output vector corrupted by an AWGN noise vector \mathbf{w}_l , where $1 \leq l \leq L$. \mathbf{w}_l is an M -dimensional Gaussian vector with zero mean and variance $\sigma^2 = (1/2)N_0$ in each real dimension. The vector representation of (4) for a single user is then

$$\mathbf{r}_l = \mathbf{s} + \mathbf{w}_l, \quad 1 \leq l \leq L. \quad (6)$$

Due to the orthogonality of the M -dimensional signal, the received vector \mathbf{r} can be defined as $\mathbf{r} = [\mathbf{r}_1, \dots, \mathbf{r}_L]$ with the probability density function (PDF) conditioned on \mathbf{s}_m as the transmitted signal given by

$$\begin{aligned} p(\mathbf{r}|\mathbf{s}_m) &= \prod_{l=1}^L \left[\left(\frac{1}{\pi N_0} \right)^{M/2} \left(\prod_{\substack{j=1 \\ j \neq m}}^M e^{-r_{lj}^2/2\sigma^2} \right) e^{-(r_{lm} - \sqrt{E_g})^2/N_0} \right] \\ &= \left(\frac{1}{\pi N_0} \right)^{ML/2} e^{-\sum_{l=1}^L \sum_{j=1, j \neq m}^M (r_{lj}^2/2\sigma^2)} \\ &\quad \times e^{-\sum_{l=1}^L (r_{lm} - \sqrt{E_g})^2/N_0}, \end{aligned} \quad (7)$$

where $r_{lj} = w_{lj}$ for $j \neq m$, $r_{lm} = \sqrt{E_g} + w_{lm}$, and w_{lj} is AWGN with zero mean and variance $\sigma^2 = (1/2)N_0$ in each real dimension.

From [13, 14, 16], the channel capacity with input signals restricted to an equiprobable M -ary signal constellation, and

no restriction on the channel output, is given by

$$\begin{aligned} C &= \log_2 M - \frac{1}{M} \sum_{m=1}^M \int \int_{\mathbf{r}} p(\mathbf{r}|\mathbf{s}_m) \log_2 \left(\frac{\sum_{n=1}^M p(\mathbf{r}|\mathbf{s}_n)}{p(\mathbf{r}|\mathbf{s}_m)} \right) d\mathbf{r} \\ &= \log_2 M - E_{\mathbf{r}|\mathbf{s}_1} \left\{ \log_2 \left(\frac{\sum_{j=1}^M p(\mathbf{r}|\mathbf{s}_j)}{p(\mathbf{r}|\mathbf{s}_1)} \right) \right\}, \end{aligned} \quad (8)$$

where $E\{\cdot\}$ is the operator for expectation value. By substituting (7) into (8), and defining $\mathbf{v}_i = \mathbf{r}_i/\sigma$, the channel capacity for an M -ary PPM UWB system over an AWGN channel with receive diversity order L can be written as

$$\begin{aligned} C_{M\text{-PPM}} &= \log_2 M - E_{\mathbf{v}|\mathbf{s}_1} \left\{ \log_2 \sum_{i=1}^M \exp \left[\sqrt{\gamma} \sum_{l=1}^L (v_{il} - v_{1l}) \right] \right\} \\ &\quad \text{bits/channel use,} \end{aligned} \quad (9)$$

where $\gamma = E_g/\sigma^2$ is the channel SNR per symbol, v_{il} , $i = 2, \dots, M$, $l = 1, \dots, L$, and v_{1l} , $l = 1, \dots, L$, are Gaussian random variables with distributions $N(\sqrt{\gamma}, 1)$ and $N(0, 1)$, respectively. $N(x, 1)$ denotes a Gaussian distribution with mean x and variance 1. Monte Carlo simulations can be applied to (9) to evaluate the channel capacity of PPM over AWGN channels with diversity order L .

3.2. Channel capacity for M -ary PAM over AWGN channels

With an M -ary PAM over AWGN channels, the received signal is no longer an M -dimensional Gaussian random variable. Let s be the M -ary PAM signal input to the channel; then the output of the l th diversity channel, r_l , with AWGN w_l and $1 \leq l \leq L$ is

$$r_l = s + w_l, \quad 1 \leq l \leq L. \quad (10)$$

The received vector \mathbf{r} with receive diversity L has an L -dimensional joint Gaussian distribution with PDF conditioned on \mathbf{s}_m as the transmitted signal given by

$$p(\mathbf{r}|\mathbf{s}_m) = \prod_{l=1}^L \left[\left(\frac{1}{\pi N_0} \right) e^{-(r_l - \sqrt{E_g})^2/N_0} \right]. \quad (11)$$

By substituting (11) into (8), the channel capacity for an M -ary PAM UWB system over an AWGN channel with receive diversity order L can be written as

$$\begin{aligned} C_{M\text{-PAM}} &= \log_2 M \\ &\quad - \frac{1}{M} \sum_{k=0}^{M-1} E \left\{ \log_2 \sum_{i=0}^{M-1} \exp \left[\sum_{l=1}^L \frac{|w_l|^2 - |s_k + w_l - s_i|^2}{N_0} \right] \right\} \\ &\quad \text{bits/channel use,} \end{aligned} \quad (12)$$

where s_i is one of the M -ary PAM signals, and w_l is an AWGN with zero mean and variance $(1/2)N_0$ in each real dimension.

3.3. Capacity of an M -ary PPM/PAM UWB system under FCC part 15 rules

Due to the possible interference to other communication systems by the UWB impulses, UWB transmissions are currently only allowed on an unlicensed basis subject to FCC part 15 rules which restricts the field strength level to 500 microvolts/meter/MHz at a distance of 3 m. This gives a transmitted power constraint for an UWB system with a 1 GHz bandwidth of $P_t \leq -11$ dBm. The following relationship is obtained using a common link budget model [5, 6]:

$$\frac{\gamma}{G} \leq -11 \text{ dBm} - N_{\text{thermal}} - F - 10 \log \frac{(4\pi d)^n}{\lambda}, \quad (13)$$

where $G = N_s T_f W_p$ is the equivalent processing gain, W_p is the bandwidth of the UWB impulse related to the pulse duration T_p , F is the noise figure, N_{thermal} is the thermal noise floor, calculated as the product of Boltzman's constant, room temperature (typically 300 K), noise figure, and bandwidth, λ is the wavelength corresponding to the center frequency of the pulse, and n is the path loss exponent. It is easily shown that the maximum reliable communication distance is determined primarily by the SNR γ . Using (9), (12), and (13), the maximum reliable distance can be calculated for an M -ary PPM/PAM with receive diversity L . The relationship between system capacity and communication range, as well as the impact of receive diversity, will be demonstrated in Section 5 via Monte Carlo simulations.

3.4. Error probability of M -ary PPM over an AWGN channel

Assuming EGC is used in the receiver, the received signal can be expressed as

$$\begin{aligned} \mathbf{r} &= \sum_{l=1}^L (\mathbf{s} + \mathbf{w}_l) \\ &= L\mathbf{s} + \sum_{l=1}^L \mathbf{w}_l. \end{aligned} \quad (14)$$

For M -ary orthogonal PPM signals, the optimal receiver consists of a parallel bank of M cross-correlators as illustrated in Figure 1. Let \mathbf{h}_j , $1 \leq j \leq M$, denote the j th basis signal vector, which is the vector representation of the basic function $h_j(t)$ shown in Figure 1, defined as

$$\mathbf{h}_j = [0, \dots, 0, 1, 0, \dots, 0], \quad (15)$$

where the nonzero value 1 is in the j th dimension. Assuming \mathbf{s}_m was sent, the optimum detector makes a decision on \mathbf{s}_m in favour of the signal corresponding to the cross-correlator with the minimum Euclidean distance

$$C(\mathbf{r}, \mathbf{h}_j) = \mathbf{r} \cdot \mathbf{h}_j, \quad j = 1, 2, \dots, M, \quad (16)$$

where

$$C(\mathbf{r}, \mathbf{h}_j) = \sum_{l=1}^L w_{lj}, \quad j \neq m, \quad (17)$$

$$C(\mathbf{r}, \mathbf{h}_m) = L\sqrt{E_g} + \sum_{l=1}^L w_{lm}.$$

Thus with the optimum detector and $N_s = 1$, the demodulated signal \hat{s} is given by

$$\hat{s} = \arg \min_{s_j} \|C(\mathbf{r}, \mathbf{h}_j) - L\sqrt{E_g}\|, \quad j = 1, 2, \dots, M. \quad (18)$$

Using standard techniques [15], the average probability of a correct decision is

$$P_c = \int_{-\infty}^{\infty} \left(\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\mathbf{r}_1/\sqrt{LN_0/2}} e^{-x^2/2} dx \right)^{M-1} p(\mathbf{r}_1) d\mathbf{r}_1, \quad (19)$$

where

$$p(\mathbf{r}_1) = \frac{1}{\sqrt{\pi LN_0}} \exp \left(-\frac{(\mathbf{r}_1 - L\sqrt{E_g})^2}{LN_0} \right). \quad (20)$$

Finally, the probability of a symbol error for an M -ary PPM is

$$P_M = 1 - P_c. \quad (21)$$

3.5. A union bound on the probability of error for M -ary PPM

Since the probability of error expressions based on (19) are complex and must be evaluated via numerical integration for large M , we now derive a simple upper bound on the symbol error probability. Assuming an equiprobable M -ary PPM constellation, an upper bound on the error probability of a PPM signal over an AWGN channel can be obtained:

$$P_{M|\mathbf{s}_m} = P_{M|\mathbf{s}_1} \leq P \left(\bigcup_{j=2}^M |C(\mathbf{r}, \mathbf{h}_j)| > |C(\mathbf{r}, \mathbf{h}_1)| \right). \quad (22)$$

The right-hand term of (22) is upper bounded by the union bound of the $M - 1$ events, that is,

$$\begin{aligned} P_{M|\mathbf{s}_1} &\leq P \left(\bigcup_{j=2}^M |C(\mathbf{r}, \mathbf{h}_j)| > |C(\mathbf{r}, \mathbf{h}_1)| \right) \\ &\leq \sum_{j=2}^M P(|C(\mathbf{r}, \mathbf{h}_j)| > |C(\mathbf{r}, \mathbf{h}_1)|) \\ &= [(M-1)P(|C(\mathbf{r}, \mathbf{h}_j)| > |C(\mathbf{r}, \mathbf{h}_1)|)]|_{\forall j \neq 1} \\ &= (M-1)Q \left(\sqrt{\frac{LE_g}{N_0}} \right). \end{aligned} \quad (23)$$

3.6. Error probability of M -ary PAM over an AWGN channel

For an M -ary PAM, the optimal receiver has a simpler structure than PPM with only one correlation receiver. With EGC assumed in (14) and using the standard techniques in [15], it is easy to show that the symbol error probability of M -ary PAM is

$$P_M = \frac{2(M-1)}{M} Q\left(\sqrt{\frac{6LE_g}{(M^2-1)N_0}}\right). \quad (24)$$

4. MULTIPLE ACCESS CAPACITY AND ERROR PROBABILITY

With more than one user active in the system, MAI is a factor limiting the performance and capacity, especially for a large number of users. As shown in [16], the net effect of the MAI produced by the undesired users at the output of the desired user's correlation receiver can be modeled as a zero-mean Gaussian random variable, if the number of users is large [15] or a repetition code is used with $N_s \gg 1$. In this section, multiple access capacity and error probability are investigated with PPM. The extension to PAM is straightforward. Assuming that $\delta \geq T_p$, that is, the M -ary PPM signal is an orthogonal signal with M dimensions, the capacity and error probability analysis given in Section 3 for a single user can be extended to multiple access systems by modifying the noise distribution.

4.1. Multiple access interference model

As given in (4), the received signal is modeled as

$$r(t) = \sum_{l=1}^L \left[\sum_{k=1}^K (s^{(k)}(t - \tau_{lk}))' + w_l(t) \right]. \quad (25)$$

To evaluate the average SNR, we make the following assumptions.

- $s^{(k)}(t - \tau_{lk})$, for $k = 1, 2, \dots, K$, where K is the number of active users, and the noise $w_l(t)$ is assumed to be independent.
- The time-hopping sequences $c_j^{(k)}$ are assumed to be independent and identically distributed (i.i.d) random variables uniformly distributed over the time interval $[0, N_h]$.
- All M -ary PPM signals are equally likely a priori.
- The time delay τ_{lk} is assumed to be i.i.d and uniformly distributed over $[0, T_f]$.
- Perfect synchronization is assumed at the receiver, that is, τ_{lk} is known at the receiver.

It will be shown later that we may assume that each information symbol only uses a single UWB pulse, that is, $N_s = 1$ for simplicity (and without loss of generality).

We assume that the desired user corresponds to $k = 1$. The basic functions of the M cross-correlators of the correla-

tion receiver for the l th path of user 1 are

$$h_{il}^{(1)}(t) = p(t - \delta_i - \tau_{1l}), \quad i = 1, \dots, M. \quad (26)$$

The output of each cross-correlator in the sample period $[nN_sT_f, (n+1)N_sT_f]$, where n is an integer, is

$$\hat{\mathbf{r}}_i = \sum_{j=nN_s+1}^{(n+1)N_s} \int_{(j-1)T_f}^{jT_f} r(t) h_{il}^{(1)}(t - jT_f - c_j^{(k)}T_c - \delta_i) dt, \quad (27)$$

$$i = 1, \dots, M.$$

Assuming PPM signal \mathbf{s}_m is transmitted by user 1, (27) can be written as

$$\hat{\mathbf{r}}_i = \begin{cases} LN_s A_d^{(1)} \sqrt{E_g} + W_{\text{MAI}} + W, & i = n, \\ W_{\text{MAI}} + W, & i \neq n, \end{cases} \quad (28)$$

where

$$W_{\text{MAI}} = \sum_{l=1}^L \sum_{k=2}^K \sum_{j=nN_s+1}^{(n+1)N_s} \int_{(j-1)T_f}^{jT_f} A_d^{(k)} p(t - jT_f - c_j^{(k)}T_c - \delta_{d_{lj/N_s}} - \tau_{kl}) \times p(t - \delta_i - \tau_{1l} - jT_f - c_j^{(k)}T_c) dt \quad (29)$$

is the MAI component and

$$W = \sum_{l=1}^L \sum_{j=nN_s+1}^{(n+1)N_s} \int_{(j-1)T_f}^{jT_f} n_l(t) p(t - \delta_i - \tau_{1l} - jT_f - c_j^{(k)}T_c) dt \quad (30)$$

is the AWGN component. By defining the autocorrelation function of $p(t)$ as

$$\gamma(\Delta) = \int_0^{T_f} p(t) p(t - \Delta) dt, \quad (31)$$

(29) can be written as

$$W_{\text{MAI}} = \sum_{l=1}^L \sum_{j=1}^{N_s} \sum_{k=2}^K A_d^{(k)} \gamma(\Delta_{lj}^{(k)}), \quad (32)$$

where $\Delta_{lj}^{(k)} = (c_j^{(1)} - c_j^{(k)})T_c - (\delta_i^{(1)} - \delta_{d_{lj/N_s}}^{(k)}) - (\tau_{1l} - \tau_{kl})$ is the time difference between user 1 and user k . Under the assumptions listed above, Δ can be modeled as a random variable uniformly distributed over $[-T_f, T_f]$. As in [1, 12, 17], the MAI is modeled as a Gaussian random process for the multiuser environment. Note that $N_s \gg 1$ justifies the Gaussian approximation even for a small number of users as illustrated in [17]. With the Gaussian approximation [1, 12, 17], we require the mean and variance of (28) to characterize the output of the cross-correlators.

It is easy to show that the AWGN component has zero mean and variance $LN_s N_0/2$. However, the mean and variance of the MAI component are determined by the specific pulse waveform. In this paper, we consider the received signal pulses to be differentiated Gaussian pulses, that is, Gaussian mono pulses are transmitted. Note that this pulse satisfies the relation $\int_{-\infty}^{\infty} p(t)dt = 0$, that is, no DC value appears in the power spectrum of the pulse. As in [11], the differentiated Gaussian pulse is defined as

$$w_{D\text{Gaussian}}(t) = \begin{cases} \frac{\sqrt{8}}{\sqrt{2\pi\lambda^3 T_p^6}} t e^{-t^2/\lambda T_p^2}, & -\frac{T_p}{2} \leq t \leq \frac{T_p}{2}, \\ 0, & \text{otherwise,} \end{cases} \quad (33)$$

where $\lambda = 0.0815$ is a bandwidth normalization parameter such that 99% of the pulse energy is contained in the range $-T_p/2 \leq t \leq T_p/2$. The autocorrelation function of the differentiated Gaussian pulse is then

$$\gamma_{D\text{Gaussian}}(\Delta) = \begin{cases} \left(1 - \frac{\Delta^2}{\lambda T_p^2}\right) e^{-\Delta^2/2\lambda T_p^2}, & 0 \leq |\Delta| \leq T_p, \\ 0, & \text{otherwise.} \end{cases} \quad (34)$$

Given (33) and (34), we have

$$E[\gamma(\Delta)] = \frac{1}{2T_f} \int_{-T_f}^{T_f} \gamma(\Delta) d\Delta = 0. \quad (35)$$

The mean of W_{MAI} can then be calculated as

$$\begin{aligned} E[W_{\text{MAI}}] &= E \left[\sum_{l=1}^L \sum_{j=1}^{N_s} \sum_{k=2}^K A_d^{(k)} \gamma(\Delta_{lj}^{(k)}) \right] \\ &= \sum_{l=1}^L \sum_{j=1}^{N_s} \sum_{k=2}^K E[A_d^{(k)}] E[\gamma(\Delta_{lj}^{(k)})] \\ &= 0, \end{aligned} \quad (36)$$

and the variance of W_{MAI} for differentiated Gaussian pulses is

$$\begin{aligned} \text{Var}[W_{\text{MAI}}] &= \text{Var} \left[\sum_{l=1}^L \sum_{j=1}^{N_s} \sum_{k=2}^K A_d^{(k)} \gamma(\Delta_{lj}^{(k)}) \right] \\ &= \sum_{l=1}^L \sum_{j=1}^{N_s} \sum_{k=2}^K E \left[(A_d^{(k)})^2 \right] E[\gamma^2(\Delta_{lj}^{(k)})]. \end{aligned} \quad (37)$$

On the basis that all PPM signals are equally likely a priori, we have

$$\begin{aligned} \text{Var}[W_{\text{MAI}}] &= \sum_{l=1}^L \sum_{j=1}^{N_s} \sum_{k=2}^K E \left[(A_d^{(k)})^2 \right] E[\gamma^2(\Delta_{lj}^{(k)})] \\ &= \frac{3\sqrt{\pi\lambda} T_p (K-1)}{8T_f} LN_s E_g, \end{aligned} \quad (38)$$

for differentiated Gaussian pulses. By defining the spread ra-

tio $\rho = T_f/T_p$, (38) can be written as

$$\text{Var}[W_{\text{MAI}}] = \frac{3\sqrt{\pi\lambda}(K-1)}{8\rho} LN_s E_g. \quad (39)$$

Hence the outputs of the cross-correlators for the receiver of user 1 can be modeled as independent Gaussian random variables with distribution

$$\begin{aligned} \hat{\mathbf{r}}_j &\sim \mathbf{N} \left(LN_s \sqrt{E_g}, \sigma_{\text{MAI}}^2 + \frac{LN_s N_0}{2} \right), \quad j = n, \\ \hat{\mathbf{r}}_j &\sim \mathbf{N} \left(0, \sigma_{\text{MAI}}^2 + \frac{LN_s N_0}{2} \right), \quad j \neq n, \end{aligned} \quad (40)$$

where $\sigma_{\text{MAI}}^2 = \zeta(LN_s E_g (K-1)/\rho)$ and $\zeta = 3\sqrt{\pi\lambda}/8$ for differentiated Gaussian pulses. Note that σ_{MAI}^2 increases with N_s , E_g , and the number of users K , but decreases with the spread ratio ρ .

Let $K = 1$ (single-user case, so that $\sigma_{\text{MAI}}^2 = 0$); then (40) can be written (after normalizing over $\sqrt{N_s}$) as

$$\begin{aligned} \hat{\mathbf{r}}_j &\sim \mathbf{N} \left(L\sqrt{N_s E_g}, \frac{LN_0}{2} \right), \quad j = m, \\ \hat{\mathbf{r}}_j &\sim \mathbf{N} \left(0, \frac{LN_0}{2} \right), \quad j \neq m, \end{aligned} \quad (41)$$

which gives the distribution of (14), the output of the correlation receiver for the single-user case, noting that $E_s = N_s E_g$. This justifies the assumption of $N_s = 1$ for the analysis in the single-user case.

4.2. Capacity considerations for multiple access

From (40), the information theoretic capacity for a PPM/PAM UWB system over an AWGN channel with a single user given in (9) can be extended to the multiple access case by substituting $\sigma_{\text{MAI}}^2 + N_s N_0/2$ for $\sigma^2 = N_0/2$, which gives

$$\begin{aligned} C_{M\text{-PPM}} &= \log_2 M - E_{\mathbf{v}|\mathbf{s}_1} \left\{ \log_2 \sum_{i=1}^M \exp \left[\sqrt{\gamma} \sum_{l=1}^L (v_{il} - v_{1l}) \right] \right\} \\ &\quad \text{bits/channel use,} \end{aligned} \quad (42)$$

where $\gamma = E_g/(\sigma_{\text{MAI}}^2 + N_s N_0/2)$ is the channel SNR per symbol.

Applying the link budget model given in (13) under FCC part 15 rules, the tradeoffs between number of users, reliable distance, and channel capacity can be determined. Numerical results will be presented in Section 5.

4.3. Multiple access error probability

Given the vector representation of the time-hopping multiple access PPM UWB system in (41), the error probability can be obtained from (21) by substituting $\sigma_{\text{MAI}}^2(LN_s N_0)/2$ for $\sigma^2 = LN_0/2$, giving

$$P_M = 1 - P_c, \quad (43)$$

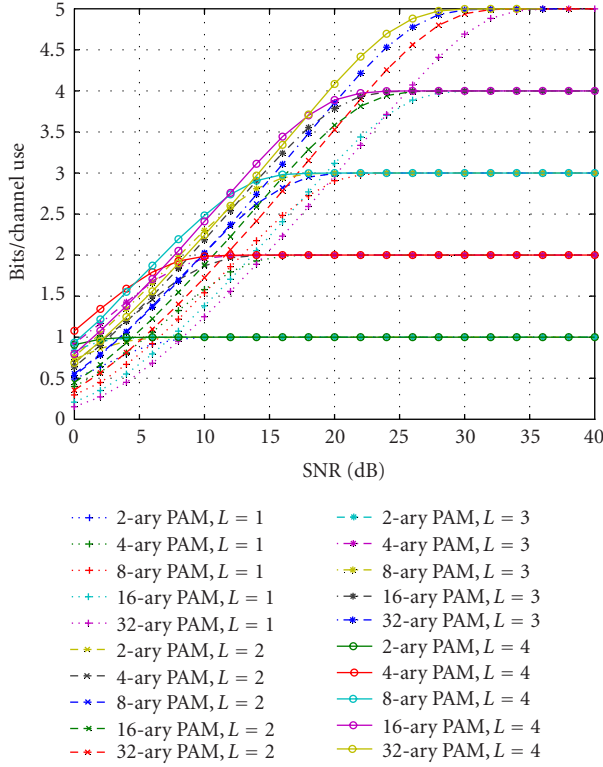


FIGURE 2: Capacity of 2-, 4-, 8-, 16-, and 32-ary PAM UWB systems with receive diversity from 1 to 4 over an AWGN channel.

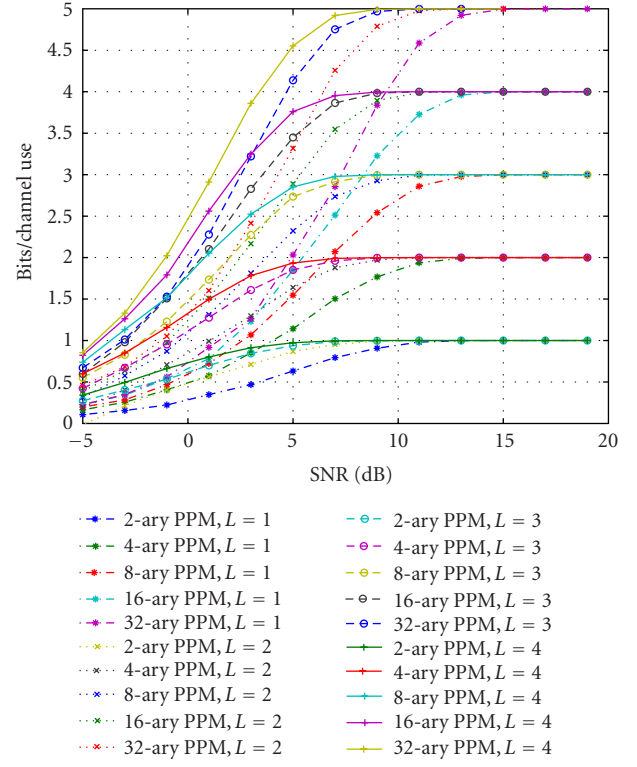


FIGURE 3: Capacity of 2-, 4-, 8-, 16-, and 32-ary PPM UWB systems with receive diversity from 1 to 4 over an AWGN channel.

where

$$P_c = \int_0^\infty \left(\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\mathbf{r}_1 / \sqrt{\sigma_{\text{MAI}}^2 + L N_s N_0 / 2}} e^{-x^2/2} dx \right)^{M-1} p(\mathbf{r}_1) d\mathbf{r}_1, \quad (44)$$

$$p(\mathbf{r}_1) = \frac{1}{\sqrt{2\pi(\sigma_{\text{MAI}}^2 + L N_s N_0 / 2)}} \times \exp \left(-\frac{(\mathbf{r}_1 - L N_s \sqrt{E_g})^2}{2(\sigma_{\text{MAI}}^2 + L N_s N_0 / 2)} \right). \quad (45)$$

The upper bound for a single-user PPM system given in (23) can then be applied to (44) to obtain

$$P_M \leq (M-1)Q \left(L N_s \sqrt{\frac{E_g}{\sigma_{\text{MAI}}^2 + L N_s N_0 / 2}} \right). \quad (46)$$

5. NUMERICAL RESULTS

In this section, some numerical results are presented to illustrate and verify the capacity and error probability expressions obtained previously.

Figure 2 shows the capacity of an M -ary PAM UWB system with a receive diversity order of 1, 2, 3, and 4 over an AWGN channel. As expected, the SNR threshold to achieve

full capacity is improved as the receive diversity increases, where the SNR is defined as E_g/N_0 . In particular, a 3 dB improvement is obtained if the diversity order is increased from 1 to 2. Approximately 2 dB in additional improvement is obtained if the diversity order is increased to 3. Figure 3 shows the capacity of an M -ary PPM UWB system with a receive diversity order of 1, 2, 3, and 4 over an AWGN channel. Similar SNR threshold improvements to those with PAM can be observed as the receive diversity increases.

Figure 4 shows the relationship between capacity and communication distance for a PPM UWB system under FCC part 15 limitations. The noise figure F is set to 10 dB and the path loss exponent n is set to 2. It can be shown that the maximum distance for reliable communications can be extended significantly with multiple receive antennas. As an example, with a 4-ary PPM, the reliable distance is extended from 90 m without receive diversity to 180 m with a receive diversity order of 4.

Figure 5 shows the performance of binary PPM and PAM UWB systems without receive diversity and with a receive diversity order of 2. A Gaussian first derivative pulse was used with a pulse width of 0.6 nanosecond and a modulation index $\delta = 0.6$ nanosecond. This shows that significant performance gains can be achieved with receive diversity. For binary PPM, there is almost a 3 dB gain with two receive antennas over one receive antenna at a BER of 10^{-3} . For binary PAM, there is also almost a 3 dB gain with two receive antennas over one receive antenna at a BER of 10^{-3} .

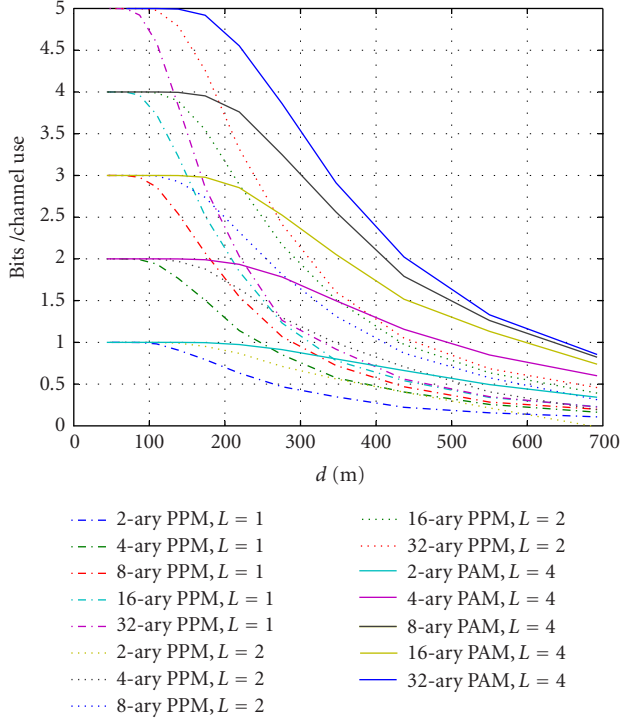


FIGURE 4: Capacity of 2-, 4-, 8-, and 32-ary PPM UWB systems versus distance with receive diversity from 1 to 4 over an AWGN channel.

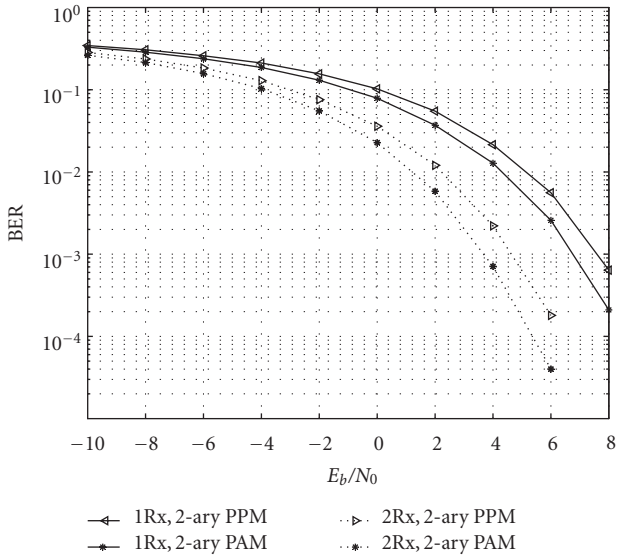


FIGURE 5: Performance of binary PPM and PAM UWB systems with receive diversity over an AWGN channel for differentiated Gaussian pulses.

The relationship between the number of users active in the network and channel capacity is illustrated in Figure 6. The processing gain was set to 32, the SNR to 5 dB, and a repetition code with $N_s = 2$ was employed. This figure shows

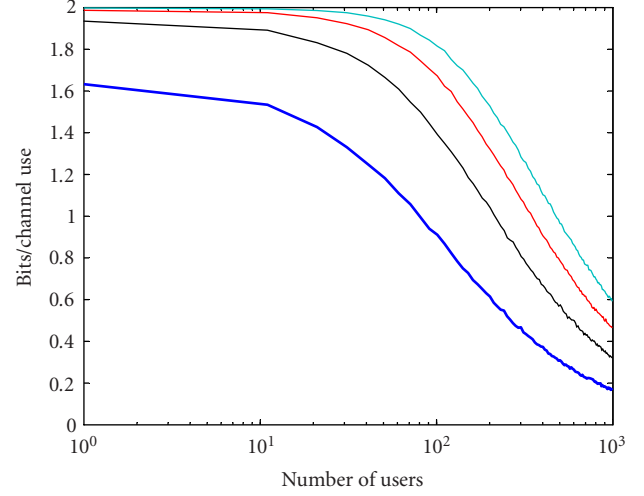


FIGURE 6: Capacity of a 4-ary PPM UWB system with $\rho = 32$, SNR = 5 dB, $N_s = 2$, and differentiated Gaussian pulses.

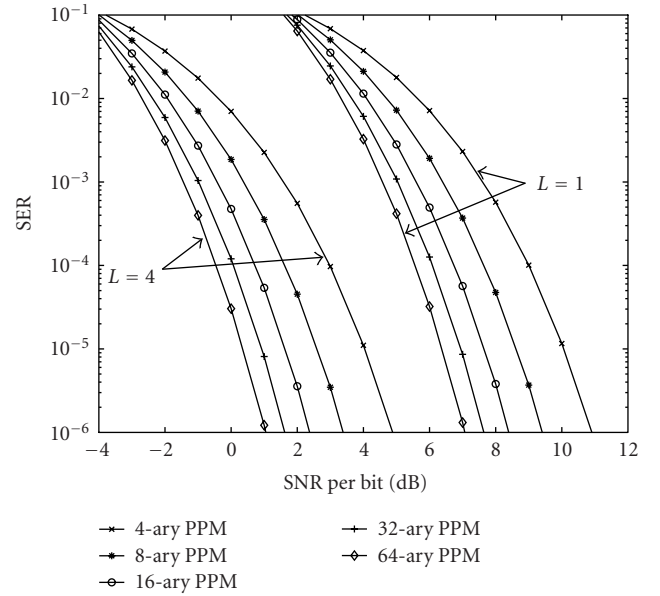


FIGURE 7: Upper bounds on the error probability for an M -ary PPM UWB system with receive diversity over an AWGN channel.

that receive diversity can significantly improve the channel capacity as expected. Due to the MAI, the achievable channel capacity decreases as the number of synchronous users increases.

Upper bounds on the error probability for M -ary PPM with a single user are shown in Figure 7. This shows that about a 6 dB gain can be obtained with a diversity order of 4. Upper bounds on the error probability for M -ary PPM

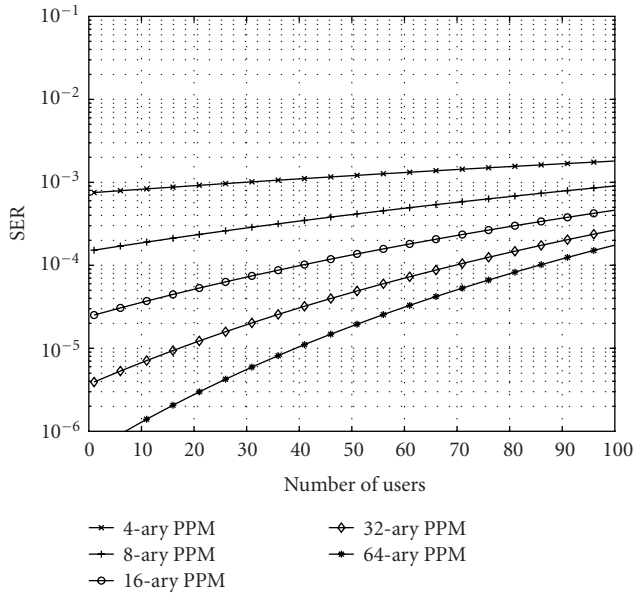


FIGURE 8: Upper bounds on the error probability for an M -ary PPM UWB system with $\rho = 500$, SNR per bit = 1 dB, $L = 2$, $N_s = 1$, and Gaussian first derivative pulses.

with multiple synchronous users are shown in Figure 8. The processing gain was set to 500, the SNR per bit was set to 1 dB, and the receive diversity order was set to 2. No repetition code was used. As expected, with a given SNR per bit, the error probability increases when the number of active users increases.

6. CONCLUSIONS

In this paper, receive diversity was proposed for PPM and PAM UWB systems to improve error performance and channel capacity, and extend the reliable communication distance. It was shown that significant improvements can be achieved due to receive diversity, which can be obtained from multiple receive antennas or resolvable multipath signals.

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